Microtorus: A High Finesse Microcavity with Whispering Gallery Modes

V.S.Ilchenko, X.S.Yao and Lute Maleki

Jet Propulsion Laboratory, CalTech

M.Gorodetsky

Physics Department, Moscow State University

Microsphere -- a low-loss photon trap, efficient optical cavity

Whispering-gallery modes - closed circular waves under total internal reflection

(Term by J.W.S.Rayleigh, analogy to acoustic modes in the gallery of St Paul cathedral)

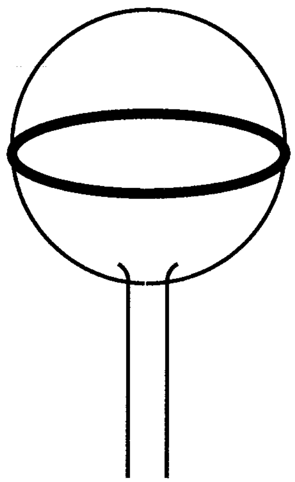
(MUST BE) Sustained in any axisymmetric dielectric body with $R \gg \lambda$

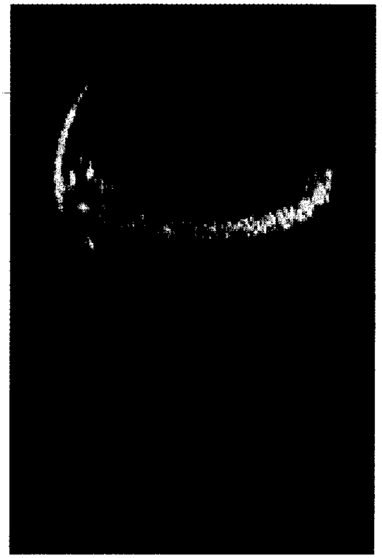
low material loss (transparent material, e.g fiber grade silica)

low bending loss ($R \gg \lambda$)

low scattering loss (TIR always under grazing incidence

+ molecular-size surface roughness)





Visualization of WG mode field by residual scattering in silica microsphere, *V.S.Ilchenko et al, Opt.Commun. 113, p.133(1994)*

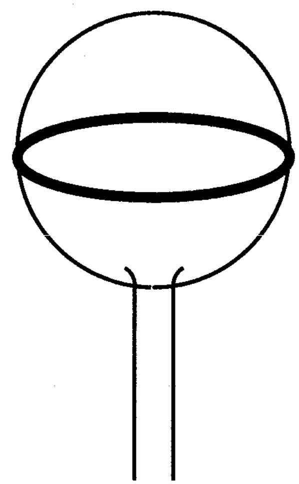
Quality-factor $Q = \lambda/\Delta\lambda_{RES}$	-	up to $\sim 10^{10}$
Photon lifetime $\tau = \lambda Q/2\pi c$	-	up to $\sim 3\mu s$
(cavity ringdown time)		
visible and near-infrared band:		*Opt.Lett. 21, p.453 (1996)*
		Opt.Lett. 23, p.247 (1998)

Why spheres?

low material loss (transparent material)

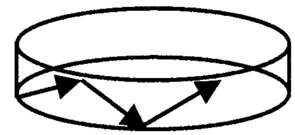
low bending loss (high-contrast boundary)

LOW SCATTERING LOSS (TIR always under grazing incidence) $\Theta \rightarrow \pi/2$; compare to disks/ rings:



$$\frac{I_R}{I_I} = e^{-(\frac{4\pi\sigma}{\lambda}\cos\Theta)^2}$$ (J.W.S.Rayleigh)

EVEN WITH MOLECULAR ROUGHNESS $\boldsymbol{\sigma}$, ONLY CURVATURE CONFINEMENT ALLOWS **Q** LIMITED BY MATERIAL ATTENUATION:



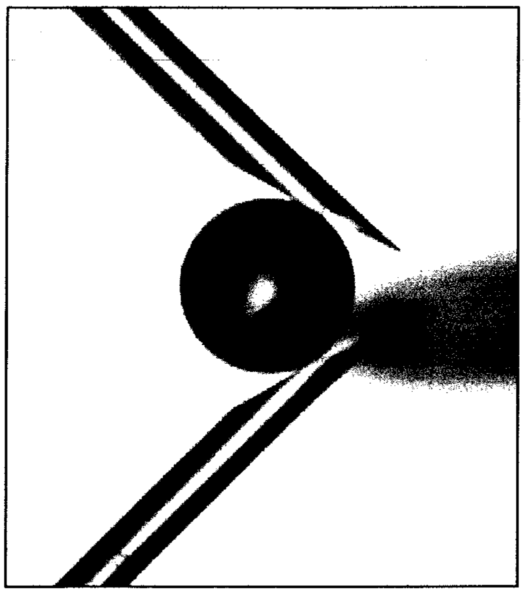
$10^8 \ldots 10^{10}$ in spheres vs. $10^3 \ldots 10^5$ in microrings !!

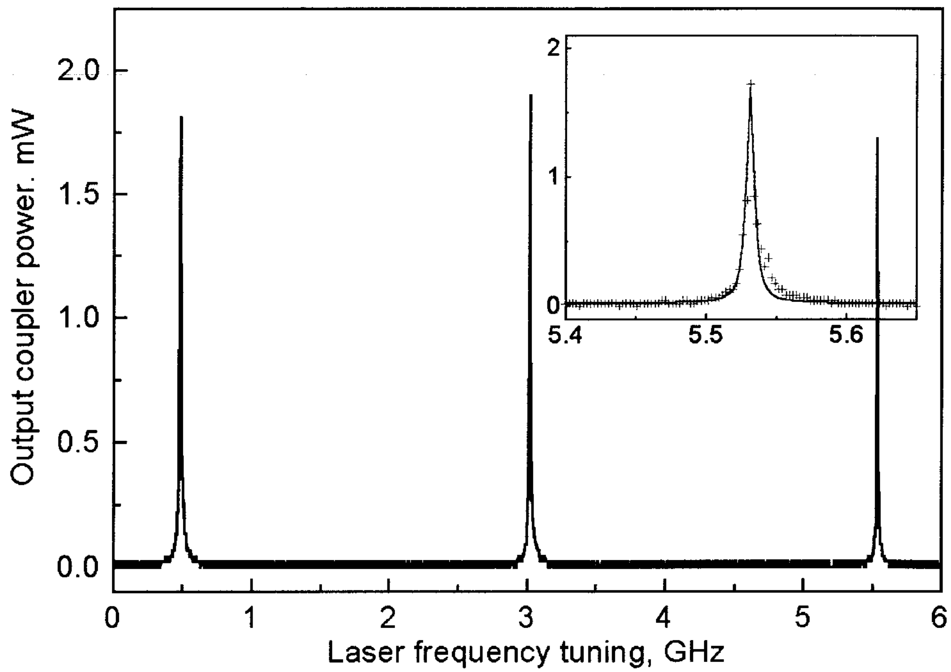
Drawback: "too many modes" compared to planar rings!

Spectrum of microspheres: Families of non-degenerate $TE(TM)_{lmq}$ modes.

"Small" FSR $\quad \nu_{lmq} - \nu_{l,m-1,q} \sim \nu \dfrac{\varepsilon^2}{2l}$ - few GHz with typical $\varepsilon^2 \sim (1\text{-}3) \times 10^{-2}$.

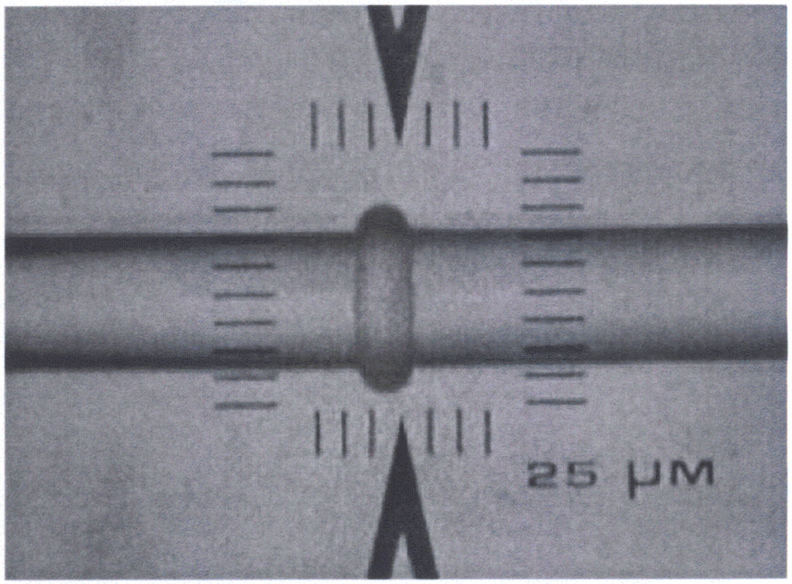
"Big" FSR $\quad \nu_{lmq} - \nu_{l-1,mq} = \dfrac{c}{2\pi n a}(t_{lq} - t_{l-1,q}) \sim \nu / l$ - few hundred GHz (few nm)

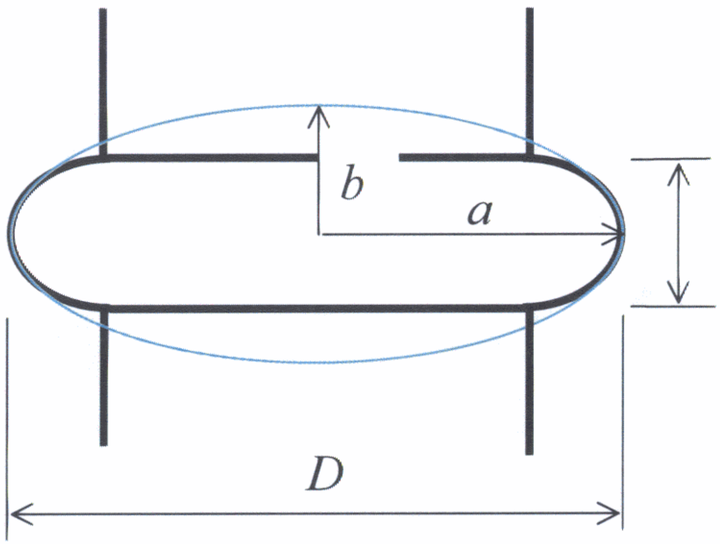




Input power 7.5…8.3mW; maximum transmission at resonance ~23.5% (fiber-to-fiber loss 6.3dB); $Q_{load} > 3 \times 10^7$ at 1550nm; sphere diameter 405μm. Unloaded $Q_o \approx 1.2 \times 10^8$ *(Opt.Lett. 24, 723 (1999))*

Novel geometry: a highly oblate spheroid, or microtorus

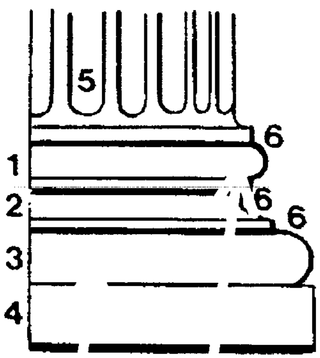




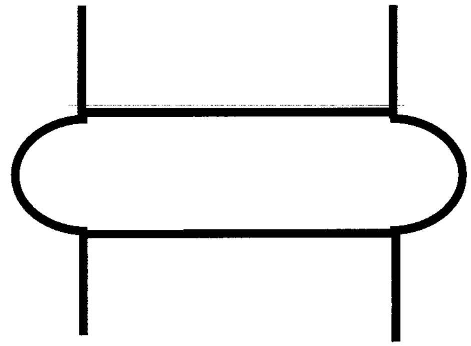
Near the symmetry plane (at the location of WG modes), toroidal surface of outer diameter D and cross-section diameter d coincides with that of the osculating oblate spheroid with large semiaxis $a = D / 2$ and small semiaxis $b = \frac{1}{2}\sqrt{Dd}$

Names

[F, seesaw] : an apparatus or structure
end is counterbalanced by the other on
or by weights
\'bā-səz\ [ME, fr.
:p, base, fr. *bainein*
a : the bottom of
support : FOUNDA-
't of a wall, pier, or
)arate architectural
)art of a complete
ı side or face of a
ıich an altitude can
)n which the figure
a bodily organ by
other more central
2 a : a main in-
latex ~> **b** : a
ıgredient (as of a
ental part of some-
the lower part of a
)int or line from which a start is made
r **b** : a line in a survey which serves as



base of a column: *1* upper torus, *2* scotia, *3* lower torus, *4* plinth, *5* shaft, *6* fillets



(Webster's New College dictionary, G & C Merriam Co. Springfiled, Mass., 1975, p.92)

Calculation of the spectrum of the dielectric spheroid

is not a trivial problem, even numerically. In "quasiclassical" approximation with assumptions:

1) a WG mode is a closed circular beam supported by TIR, 2) optical field tunnels outside at the depth $1/k\sqrt{n^2-1}$, and 3) the tangential component of E (*TE*-mode), or normal of D (*TM*-mode) is continuous at the boundary. Eigenfrequencies of high-order WG modes ($l >> 1; l \approx m$) in dielectric sphere can be approximated via solutions of scalar wave equation with zero boundary conditions, because most of the energy is concentrated in one component of the field (E_θ for *TE*-mode and E_r for *TM*-mode).

Based on above considerations, let us estimate WG mode eigenfrequencies in oblate spheroids of large semiaxis a, small semiaxis b, and eccentricity $\varepsilon = \sqrt{1-b^2/a^2}$. Since WG modes are localized the "equatorial" plane, we shall approximate the radial distribution by *cylindrical* Bessel function $J_m(n\tilde{k}_{mq}r)$ with $n\tilde{k}_{mq}a = na\sqrt{k_{lmq}^2 - k_\perp^2} \approx T_{mq}$, where $J_m(T_{mq}) = 0$ and $k_\perp$ is the wavenumber for quasiclassical solution for angular spheroidal functions. For our purposes a rough approximation is enough: $k_\perp^2 \approx \frac{2(l-m)+1}{a^2\sqrt{1-\varepsilon^2}}m$;

more rigorous consideration can follow the approach given in [*I.V.Komarov, L.I.Ponomarev, S.Yu.Slavyanov, Spheroidal and Coulomb Spheroidal Functions, Moscow,Nauka (1976) (in Russian)*]. Taking into account that $T_{mq} \approx t_{lq} - (l - m + 1/2)$, we finally obtain the following approximation:

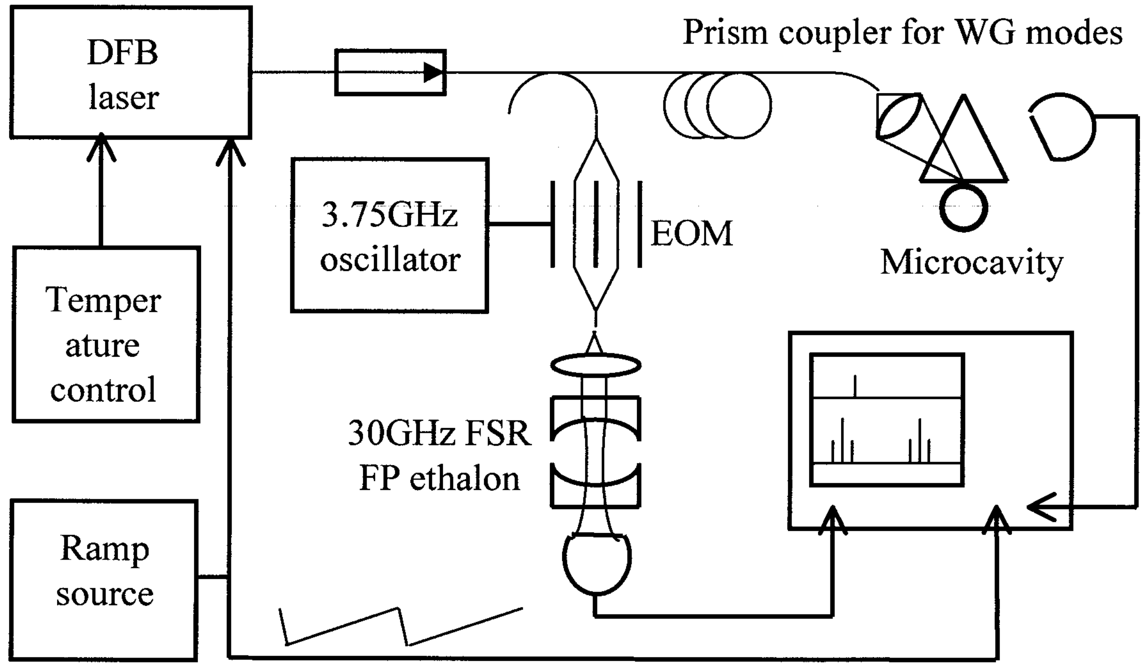
$$nk_{lmq}a - \frac{\chi}{\sqrt{n^2-1}} \approx t_{lq} + \frac{2(l-m)+1}{2}\left(\frac{1}{\sqrt{1-\varepsilon^2}} - 1\right)$$

t_{lq} -- q-th zero of the spherical Bessel function of the order l; $\chi = n$ for *TE*-mode, $\chi = 1/n$ for *TM*-mode.

1. For small eccentricities the model gives identical prediction with perturbation theory *(H.M.Lai, P.T.Leung, K.Young, P.W.Barber, S.C.Hill, Phys. Rev. A 41, 5187-5198 (1990))*
2. Discrepancy with numerical calculations is <5% in prediction of "small" FSR and <0.1% of absolute frequencies, even with $\varepsilon^2 \sim 0.8$, even small $l = 100$

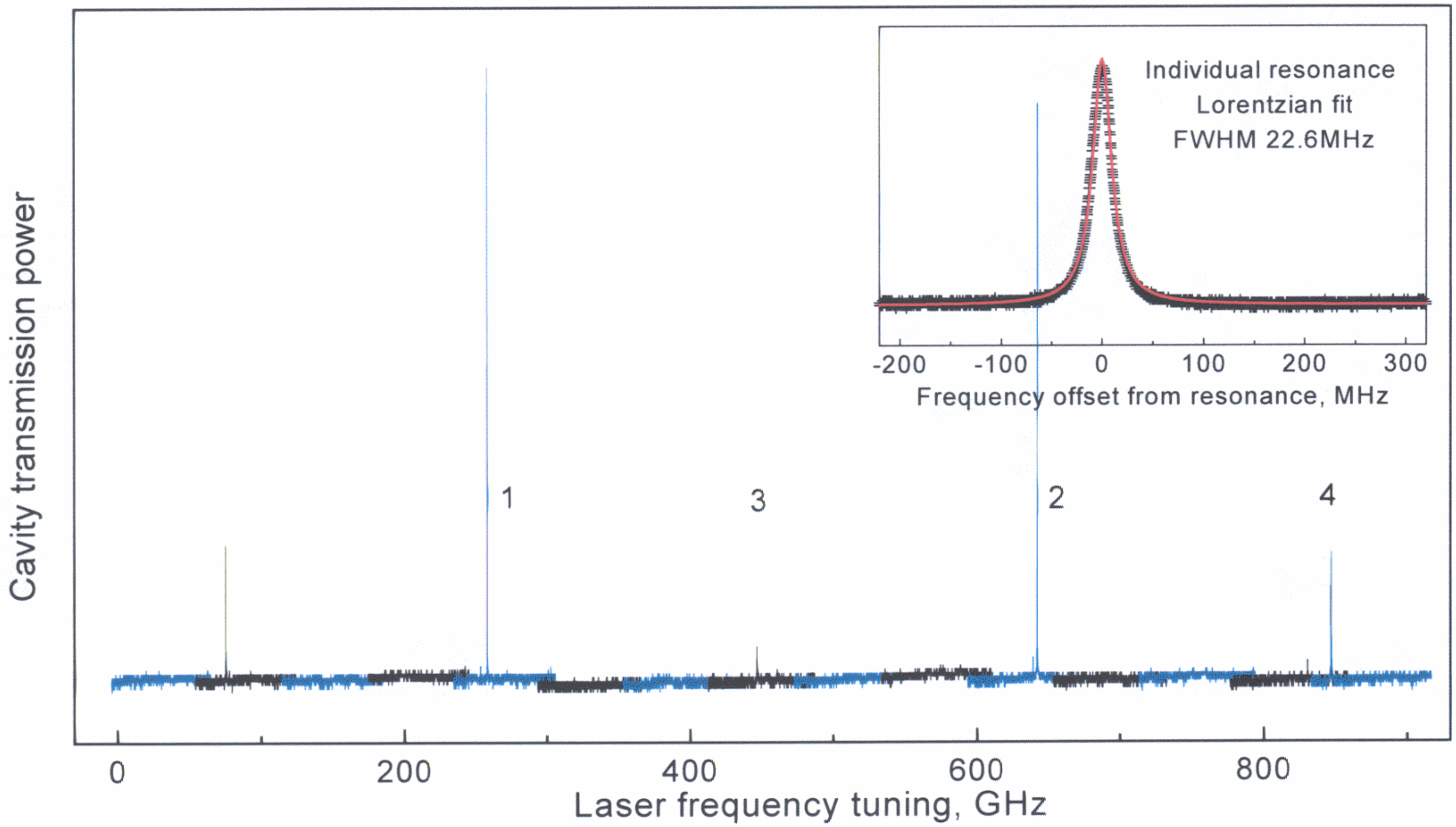
Schematic of the experimental setup

to obtain wide range (~900GHz, or 7.2nm) high-resolution spectra of WG modes in microcavity



Spectrum of *TE* whispering-gallery modes in spheroidal dielectric microcavity

$D = 2a = 165\mu m$; $d = 42\mu m$; $2b = 83\mu m$. Free spectral range (between largest peaks 1 and 2) 383.7GHz (3.06nm) near central wavelength 1550nm. Individual resonance bandwidth 23MHz (loaded Q = 8.5×10^6). Finesse $F = 1.7\times10^4$



CONCLUSIONS

1. It seems indeed we can combine small size, ultra-high-Q with "nice" FP-like spectrum: true finesse $10^4 \ldots 10^6$ becomes available in microcavities as opposed to supermirror FPs

2. Host of potential applications is significant